On the Use of Heat Belts for Energy Integration Across Many Plants in the Total Site

Miguel Bagajewicz* and Hernán Rodera

School of Chemical Engineering and Material Science, University of Oklahoma, Norman, OK 73019-0628, USA

Indirect heat integration across plants using intermediate fluids is, in many cases, a preferable alternative to direct integration. Previous work has discussed the use of independent circuits capable of transferring the maximum possible heat. Piping cost and the location of the plants were proven an important factor when intermediate fluids are used. This paper analyzes the concept of a 'heat belt' that consists of a single-pipe circuit used to extract heat from and release it to the plants. Since a general model is MINLP, the analysis focuses on the particular case of three plants for which an MILP model can be formulated.

L'intégration indirecte de la chaleur entre usines utilisant des fluides intermédiaires est, dans bien des cas, une option préférable à l'intégration directe. Le recours à des circuits indépendants capables de transférer le maximum de chaleur a été examinéantérieurement. Le coût des conduites et l'emplacement des usines s'avèrent un facteur important lorsque des fluides intermédiaires sont utilisés. On analyse dans cet article le concept de 'ceinture de chaleur' qui consiste en un circuit à tuyau unique servant à extraire la chaleur des usines pour l'envoyer vers d'autres usines. Puisque le modèle général est de type MINLP, l'analyse examine le cas particulier de 3 usines pour lesquelles un modèle MILP peut être formulé.

Keywords: heat integration, energy savings, total site integration, heat belt.

Indirect heat integration across plants via intermediate fluids like steam or dowtherms is an alternative to direct integration using process streams. Early studies by Dhole and Linnhoff (1992) and Hui and Ahmad (1994) on total site heat integration helped to determine the levels of generation of steam necessary to indirectly integrate different processes. As generation and use of steam have to be performed at a fixed temperature level, opportunities for integration are lost. Rodera and Bagajewicz (1999a) demonstrated the magnitude of these losses by developing targeting procedures for direct and indirect integration in the special case of two plants. Application of pinch analysis showed that the heat transfer effectively leading to energy savings occurs at temperature levels between the pinch temperatures of both plants. In some other cases, however, heat transfer in regions that are not between the plant pinch temperatures is required to attain maximum savings (assisted heat integration).

Bagajewicz and Rodera (2000) presented generalized mathematical models extending the results originally developed for two plants (Rodera and Bagajewicz, 1999a) to the case of multiple plants. In addition, Rodera and Bagajewicz (1999c) showed preliminary results of the application of these models where an MILP model is proposed to locate the intermediate fluid circuits that perform the indirect integration. Moreover, the optimal location of these circuits that allows flexibility of operation was added to this formulation to consider cases where any of the plants was shut down.

A reduction of the piping and pumping costs can be expected if a single pipe collects heat from and delivers it to the plants. In cases in which independent circuits transfer heat from the same plant to many other plants, a pair of pipes has to be used for each transfer. Additionally, more heat exchangers may be necessary. The relative location of each of the plants to the others also plays an important role. In many aspects, simplicity then can be obtained by using a single 'heat belt' system that takes advantage of the existing location of the plants. Rodera and Bagajewicz (1999b) presented a case study on integration between two plants that supports this idea. In this study, piping and pumping costs are such that the use of one circuit instead of two is more economical, even though one circuit does not achieve all the possible energy savings.

In this paper, the new concept of the heat belt is introduced as an alternative to the use of independent or multi-operation circuits. The concept is derived by restricting multi-operation circuits to the use of a single pipe arrangement, as presented by Bagajewicz and Rodera (2000). The MINLP model that locates the heat belt for the special case of three plants is then introduced. Linearization of this model is possible as only bilinearities consisting of the product of a binary variable and a continuous variable are present. Examples showing the different features of this approach are presented.

*Author to whom correspondence may be addressed. E-mail address: bagajewicz@ou.edu

The Concept of Heat Belt

In a recent article, Bagajewicz and Rodera (2000) suggested the use of a heat belt to help gather heat from some plants and deliver it to others. It was speculated that this could save some piping/pumping costs and possibly resolve the issue of flexibility, as

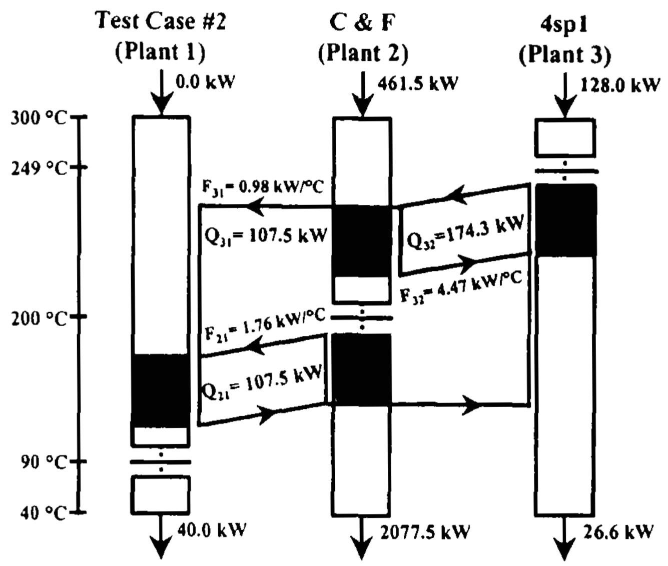

Figure 1. Multi-operation circuit.

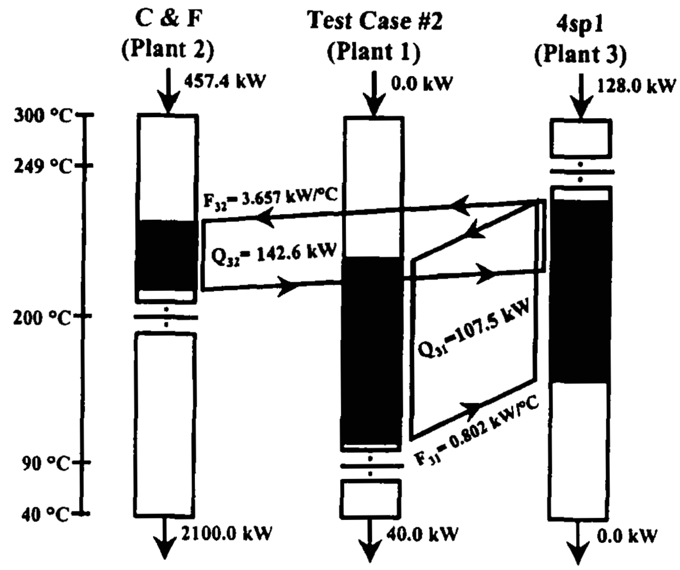

Figure 2. Independent circuits for rearranged plants.

multiple alternative solutions could be included in the heat belt. This would allow for maximum efficiency in multiple scenarios of plants shutting down and/or changing throughput.

To attain the maximum possible savings in any operating condition, Bagajewicz and Rodera (2000) solved their targeting model for circuit location considering all of the possible operational modes. These modes are the instances in which two or more plants are considered to be in operation. If the restriction of the use of a single pipe arrangement is imposed, sometimes alternative circuits can be obtained that use less piping. Figure 1, taken from Bagajewicz and Rodera (2000), shows an example of the circuit required for this task established across three plants. This circuit works as two independent circuits, one between plant 2 and plant 1 and the other between plant 3 and plant 1 when integration of the total site is present. The external circuit is available for the case in which plant 2 is shut down, and it allows the transfer of heat from plant 3 to plant 1.

One of the characteristics of the scheme in Figure 1 is that certain parts of the two circuits are common. Thus, it is practical to join these circuit parts in a single pipe by restricting the extended MILP model. The new idea is that the heat transfer circuits can be thought of as a heat-belt circuit from which the different plants take and/or discharge fluid to extract and/or release heat. In this new approach, mixing plays an important role. As is immediately apparent, the position of the plants sometimes does not suggest a heat belt. To illustrate this, consider Figure 2 in which the plants of Figure 1 have been rearranged. The use of independent circuits no longer suggests the location of the heat belt.

In the analysis that follows, *one* of the possible heat belts that can be established across a set of *n* plants is discussed. This version of a heat belt, is formally introduced by the use of Figure 3, in which *n* plants are aligned. The heat belt is a fixed circuit that has two main lines, the top line and the bottom line. Branches split and mix with the main lines as the heat belt passes by the plants. For each of the plants, two branches are present: 1) the hot line F_j^H, from which the plant receives heat from the heat belt, and 2) the cold line F_j^C, from which the plant delivers heat to the heat belt. Extreme plants have only one branch, because plant 1 can only receive heat from the heat belt (heat leading to savings is transported only from the right to the left), and plant *n* can only deliver heat to the heat belt.

The top line starts at the extreme right with the amount of heat collected by the cold branch from plant *n*. As it passes through each of the plants, hot fluid branches are split to deliver heat. Each split is followed by the mixing of the remaining hot fluid with cold branches that have collected heat from the plants. Similarly, the bottom line starts at the extreme left, following the hot branch that has delivered heat to plant 1. Cold branches are split to collect heat from each of the plants. Splits are followed by mixers in which cold branches are combined with hot branches that have delivered heat.

The heat belt considered in this article relates only to plants that are aligned as shown in Figure 3 (i.e., an arrangement in which the plant are located side by side one another in a sequence). However, other arrangements are possible (plants in the vertex of a triangle or a rectangle), and therefore, other definitions for the heat belt might follow. Analysis of these other alternatives is left for future work.

Basic concepts referring to heat integration across plants in the total site were introduced by the authors in previous articles

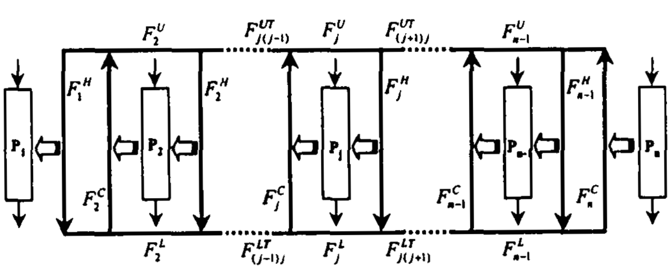

Figure 3. General heat belt.

(Rodera and Bagajewicz, 1999a; Bagajewicz and Rodera, 2000). These concepts are briefly reviewed in Appendix 1, and some features of the heat belt are immediately apparent when they are considered:

- Because of the heat belt definition, it is not possible to transfer assisted heat in the direction opposite to the effective heat (i.e. from left to right).
- The plant with the highest pinch temperature can never receive effective heat. However, in principle it may receive assisted heat in the same direction as the effective heat from other plants. Thus, if the plant is located on the extreme left (plant 1), only the intervals above its pinch temperature can receive assisted heat in the same direction as the effective heat. If one can establish *a priori* that assisted heat is not needed, then the plant with the highest pinch temperature located on the extreme left cannot participate in the heat belt.
- Similarly, the plant with the lowest pinch temperature can never deliver effective heat. In principle, it may give assisted heat in the same direction as the effective heat to other plants. If it is located on the extreme right (plant n), only the intervals above its pinch temperature can receive heat, and in the absence of assisted heat in the same direction as the effective heat, a plant located on the extreme right cannot participate in the heat belt.

For the special case of three plants and no assisted heat in either direction, three geographical location cases can be considered as candidates for a heat belt. These cases of relative location are described by the following situations:

- The pinch temperatures are increasing from left to right;
- The plant with the intermediate pinch temperature is located either on the extreme left or on the extreme right, while the plant having the highest pinch temperature is never located on the extreme left and the plant with the lowest pinch temperature is never on the extreme right.

A mathematical programming model for the case of three plants is presented next.

MILP for the Three Plants Case

Figure 3 presents general definitions for the different flowrate variables. In order to establish the upper limit in the temperature intervals for the hot and cold branches, binary variables Y_{ij}^{UH} and Y_{ij}^{UC} are defined. Similarly, binary variables Y_{ij}^{LH} and Y_{ij}^{LC} are defined to establish the lower limit in the temperature intervals for their respective branches. In turn, these binary variables determine whether or not there is heat transfer with the heat belt by setting the continuous variables Z_{ij}^{H} and Z_{ij}^{C} to either one or zero. Finally, the variable X_j^H represents the enthalpy of the hot branches before delivering heat, and X_j^C represents the enthalpy of the cold branches before receiving heat. The following MINLP model establishes the optimal heat-belt circuit for the special case of a given set of three plants:

$$\mathbf{P1} = Min\left(\delta_o^{\mathrm{I}} + \delta_o^{\mathrm{II}}\right) \tag{1}$$

s.t.

$$\left.\begin{array}{l}
\delta_o^{I} = \hat{\delta}_o^{I} - \sum_{i=1}^{m} q_{i,1}^{H} \\
\delta_o^{II} = \hat{\delta}_o^{II} - \sum_{i=1}^{m} q_{i,2}^{H} \\
\left.\begin{array}{l}
\delta_i^{I} = \delta_{i-1}^{I} + q_i^{I} + q_{i,1}^{H} \\
\delta_i^{II} = \delta_{i-1}^{II} + q_i^{II} + q_{i,2}^{H} - q_{i,2}^{C} \\
\delta_i^{III} = \delta_{i-1}^{III} + q_i^{III} - q_{i,3}^{C}
\end{array}\right] \forall i = 1,\ldots,m \\
\delta_m^{II} = \hat{\delta}_m^{II} - \sum_{i=1}^{m} q_{i,2}^{C} \\
\delta_m^{III} = \hat{\delta}_m^{III} - \sum_{i=1}^{m} q_{i,3}^{C}
\end{array}\right\} \tag{2}$$

$$\left.\begin{array}{l}
F_1^H = F_2^U + F_2^C \\
F_2^U + F_2^H = F_3^C \\
F_3^C = F_2^H + F_2^L \\
F_2^C + F_2^L = F_1^H
\end{array}\right\} \tag{3}$$

$$\left.\begin{array}{lll}
q_{ij}^{H} - U_{ij}^{H} Z_{ij}^{H} \le 0 & \forall i = 1,\ldots,m & \\
Z_{ij}^{H} = Y_{(i-1)j}^{UH} & i = 1 & \\
Z_{ij}^{H} = Z_{(i-1)j}^{H} + Y_{(i-1)j}^{UH} - Y_{(i-1)j}^{LH} & \forall i = 2,\ldots,m & j = 1,2 \\
\sum_{i=0}^{m-1} Y_{ij}^{UH} = 1 & & \\
\sum_{i=1}^{m} Y_{ij}^{LH} = 1 & &
\end{array}\right\} \tag{4}$$

$$\left.\begin{array}{lll}
q_{ij}^{C} - U_{ij}^{C} Z_{ij}^{C} \le 0 & \forall i = 1,\ldots,m & \\
Z_{ij}^{C} = Y_{(i-1)j}^{UC} & i = 1 & \\
Z_{ij}^{C} = Z_{(i-1)j}^{C} + Y_{(i-1)j}^{UC} - Y_{(i-1)j}^{LC} & \forall i = 2,\ldots,m & j = 2,3 \\
\sum_{i=0}^{m-1} Y_{ij}^{UC} = 1 & & \\
\sum_{i=1}^{m} Y_{ij}^{LC} = 1 & &
\end{array}\right\} \tag{5}$$

$$\left.\begin{array}{l}
X_1^H = \sum_{i=0}^{m-1} \left(Y_{i,2}^{UC} F_2^C T_i + Y_{i,3}^{UC} F_3^C T_i\right) - X_2^H \\
X_2^H = \sum_{i=0}^{m-1} Y_{i,3}^{UC} F_2^H T_i \\
X_2^C = \sum_{i=1}^{m} Y_{i,1}^{LH} F_2^C T_i \\
X_3^C = \sum_{i=1}^{m} \left(Y_{i,1}^{LH} F_1^C T_i + Y_{i,2}^{LH} F_2^C T_i\right) - X_2^C
\end{array}\right\} \tag{6}$$

$$
\left.\begin{aligned}
&X_j^H - \sum_{i=1}^{m} Y_{i,j}^{LH} F_j^H T_i = \sum_{i=1}^{m} q_{i,j}^H \\
&\sum_{i=0}^{m-1} Y_{i,j}^{UC} F_j^C T_i - X_j^C = \sum_{i=1}^{m} q_{i,j}^C
\end{aligned}\right\} \quad (7)
$$

$$
\left.\begin{aligned}
&F_1^H \sum_{i=1}^{r} Z_{i,1}^H \Delta T_i + \sum_{i=0}^{r-1} Y_{i,1}^{UH}\left(X_1^H - F_1^H T_i\right) \geq \sum_{i=1}^{r} q_{i,1}^H && \forall r = 1,\ldots,(m-1) \\
&F_1^H \sum_{i=1}^{m} Z_{i,1}^H \Delta T_i + \sum_{i=0}^{m-1} Y_{i,1}^{UH}\left(X_1^H - F_1^H T_i\right) = \sum_{i=1}^{m} q_{i,1}^H && \\
&X_1^H - F_1^H T_i \geq 0 && \forall i = 0,\ldots,m-1
\end{aligned}\right\} \quad (8)
$$

$$
\left.\begin{aligned}
&F_2^H \sum_{i=1}^{r} Z_{i,2}^H \Delta T_i \geq \sum_{i=1}^{r} q_{i,2}^H && \forall r = 1,\ldots,(m-1) \\
&F_2^H \sum_{i=1}^{m} Z_{i,2}^H \Delta T = \sum_{i=1}^{m} q_{i,2}^H &&
\end{aligned}\right\} \quad (9)
$$

$$
\left.\begin{aligned}
&F_3^C \sum_{i=r}^{m} Z_{i,3}^C \Delta T_i + \sum_{i=r}^{m} Y_{i,3}^{LC}\left(F_3^C T_i - X_3^C\right) \geq \sum_{i=r}^{m} q_{i,3}^C && \forall r = 2,\ldots,m \\
&F_3^C \sum_{i=1}^{m} Z_{i,3}^C \Delta T_i + \sum_{i=2}^{m} Y_{i,3}^{LC}\left(F_3^C T_i - X_3^C\right) = \sum_{i=1}^{m} q_{i,3}^C && \\
&F_3^C T_i - X_3^C \geq 0 && \forall i = 2,\ldots,m
\end{aligned}\right\} \quad (10)
$$

$$
\left.\begin{aligned}
&F_2^C \sum_{i=r}^{m} Z_{i,2}^C \Delta T_i \geq \sum_{i=r}^{m} q_{i,2}^C && \forall r = 2,\ldots,m \\
&F_2^C \sum_{i=1}^{m} Z_{i,2}^C \Delta T_i = \sum_{i=1}^{m} q_{i,2}^C &&
\end{aligned}\right\} \quad (11)
$$

$$
\begin{aligned}
&\delta_i^I, \delta_i^{II}, \delta_i^{III}, q_{ij}^H, q_{ij}^C, F_j^H, F_j^C, F_2^U, F_2^L, X_j^H, X_j^C, Z_{ij}^H, Z_{ij}^C, \geq 0 \\
&Y_{ij}^{UH}, Y_{ij}^{LH}, Y_{ij}^{UC}, Y_{ij}^{UC} \in \{0,1)
\end{aligned} \quad (12)
$$

The objective function (1) minimizes the summation of the heating utilities of the first two plants because no heating utility savings are possible in plant 3 (no assisted heat in either direction is considered). Reduction of these utilities is attained by the heat transfer from the heat belt to the respective plants. In turn, the respective cooling utilities are reduced by the transfer of heat from the plants to the heat belt (with the exception of plant 1). Equations (2) represent the balances established to allow all the abovementioned heat transfer. Flowrate balances are established by Equations (3) in all the nodes in which the heat belt branches are split or mixed. Equations (4) and (5) establish the intervals in which heat transfer from the plants to the heat belt and from the heat belt to the plants is allowed. The formulation of these equations, which are equivalent to the ones used by Rodera and Bagajewicz (1999a) to establish intervals in which a circuit spans across two plants, can be found Appendix 2. Expressions for all the enthalpies as a function of the respective flowrates and fixed

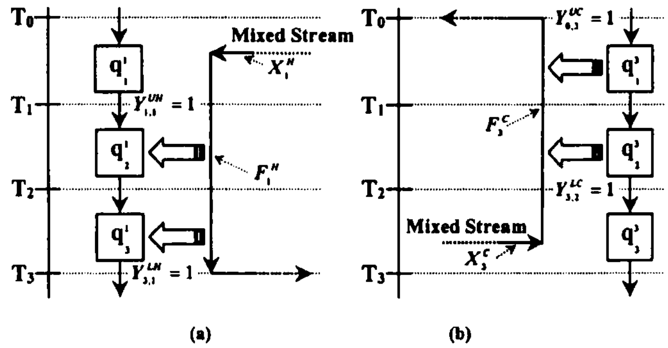

Figure 4. Enthalpies of the heat belt.

interval temperatures are given by Equations (6). Equations (7) guarantee that the heat transfer to and from the heat belt equals the available enthalpy difference.

The nature of the thermodynamic feasibility constraints found in Equations (8) through (11) is explained next. Similar equations are used by Rodera and Bagajewicz (1999a) for the feasible location of circuits between two plants. Due to the possibility of mixing of streams in the heat belt, an extra positive term is needed in the restrictions imposed in Equations (8) and (10). In the case of Equations (8), the enthalpy of the hot branch just before delivering heat to plant 1 (X_1^H) is obtained by mixing. As Figure 4a shows, this enthalpy is located somewhere within the fixed temperature interval in plant 1 immediate above the temperature level at which the heat cascade starts receiving heat. Therefore, the difference between the temperature that the hot branch has (obtained by mixing) and the temperature at which the heat cascade of plant 1 starts receiving heat is not available for heat transfer. The extra term added to the feasibility constraint in Equations (8) makes possible this temperature difference. This term must always be positive, because the temperature of the hot branch must be greater than or equal to the upper temperature of the starting interval. Similar analyses can be done for the case of Equations (10), in which the enthalpy of the cold branch before receiving heat from plant 3 (X_3^C) is obtained by mixing. Figure 4b shows the location within the fixed temperature interval in plant 3 immediate below the temperature at which the cold branch starts receiving heat.

As only unassisted heat integration is considered, the region in which the heat belt is established lies between the extreme pinch temperatures. Therefore, the zone above the highest pinch temperature and the zone below the lowest pinch temperature do not need to be considered. For the transfer of heat to and from the intermediate fluid to be possible, a shift of scales similar to the one implemented in Rodera and Bagajewicz (1999a) is necessary. Consider the three-plant arrangement sorted by increasing pinch temperatures shown in Figure 5. The cold branches located below the plant pinch temperature zone are heated up, and they represent cold streams that receive heat from the hot streams located in this zone. In turn, the hot branches located above the plant pinch temperature zone are cooled down, and they represent hot streams delivering heat to cold streams located in this zone. The hot scales of the zone of plants that deliver heat (i.e. below their pinch temperatures) and the cold scales of the zone of plants that receive heat (i.e. above their pinch temperatures) must coincide in order to

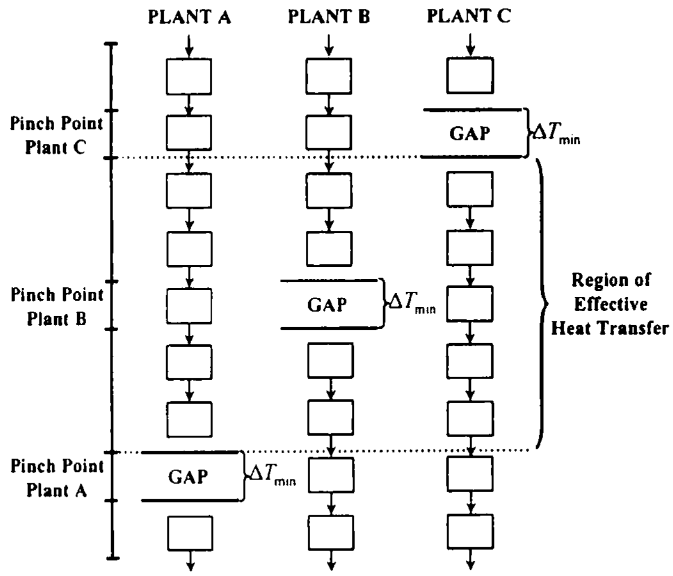

Figure 5. Scales shifting below the plant pinch temperatures.

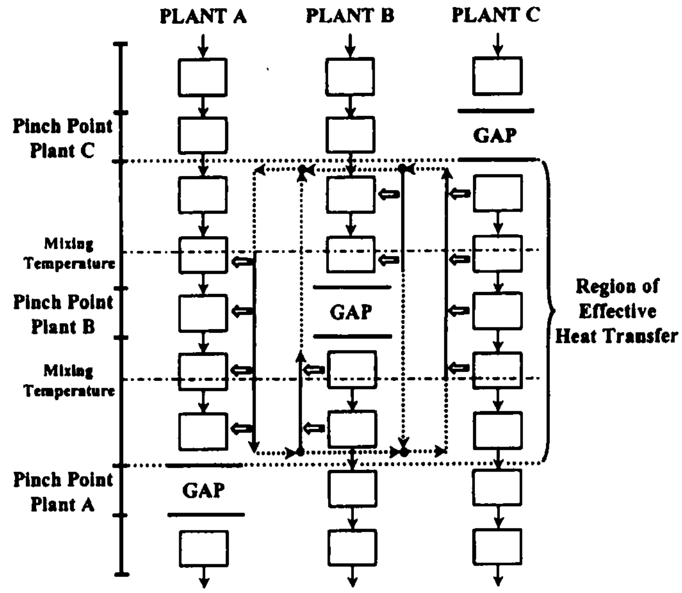

Figure 6. Heat belt for 1st case.

establish the heat belt. Therefore, the hot and cold scales below the pinch temperatures of the plants are shifted downward ΔT_{min} degrees. The process of moving these scales down generates gaps as shown in Figure 5. The shifting does not depend on the location of the plants relative to each other. The sequence can be altered, but the shift of the scales remains the same. That is, the scales of all the plants below their pinch temperatures are shifted downwards.

Model P1 is MINLP because of the presence of bilinear terms that include the product of a continuous variable times a binary variable. These terms are replaced by their equivalent set of linear inequalities, and an MILP model is obtained. Equations (6) to (10) render replacements similar to those employed in Rodera and Bagajewicz (1999).

Heat Belt Solutions

As previously mentioned, for the particular case of three aligned plants, three locations of one plant relative to the others are possible. Consider the arrangement with the plants arranged with increasing pinch temperature location from left to right (1st case). The heat belt reduces to the one shown in Figure 6. Filled lines represent the temperature ranges for hot and cold branches in which heat transfer takes place. Therefore, the dotted parts of the circuit preserve their initial temperatures. A special case of these general heat belt are the independent circuits between plants that are located one by the side of the other. Notice that in the case of Figure 6, if the heat transfer is only allowed between pinches and the branches of the heat belt that bypass plant 2 are eliminated independent circuits between plant 3 and plant 2 and plant 2 and plant 1 are obtained.

Now consider the arrangement in which the plant with the intermediate pinch temperature is located first on the left, followed by the plant with the lowest pinch temperature and the plant with the highest pinch temperature (2nd case). The heat belt for this case is presented in Figure 7. Notice that no cold branch is possible on the left side of the plant with the lowest pinch temperature.

In the last case (Figure 8), the sequence of Figure 6 is altered by locating the plant with the highest pinch temperature in the middle and the plant with the intermediate pinch temperature last in the sequence (3rd case). Because of its position, no hot branch is possible on the right side of the plant with the highest pinch temperature.

Analysis Using Composite Curves

In the analysis that follows, the heat cascade diagram previously presented to introduce the heat belt is compared with the traditional diagram that makes use of the Grand Composite Curves of the processes. Adopting the hot streams scale of temperatures, consider the Grand Composite Curves for three plants shown in the broken lines of Figure 9a. Notice the separation between the hot and the cold Grand Composite Curves that represent the gap required to make possible the indirect heat transfer between the plant and the heat belt. These curves correspond to the 1st case arrangement (increasing pinch temperatures order). The continuous lines represent the independent circuits that are obtained when only transfer between pinches is allowed and the branches that bypass plant 2 are eliminated. Ahmad and Hui (1991) showed a similar diagram for the two plants case. They explained that when heat transfer is not restricted to a constant temperature (e.g. steam as in Dhole and Linnhoff, 1992), but has a sloping profile (i.e., the intermediate fluid of the heat belt), transfer from the cold Grand Composite Curve of one plant to the hot Grand Composite Curve of the other could extend until the pinch of the latter is reached. The argument is clearly illustrated if the Grand Composite Curve of the heat receiving process is inverted (Ahmad and Hui, 1991).

When the general heat belt is considered, a diagram like the one shown in Figure 9b is obtained. Now the heat transfer from the cold Grand Composite Curve of plant 3 to the heat belt extends beyond the pinch point of plant 2. The splitter located before the heat-belt hot branch starts delivering heat to plant 2 (splitter 1), generates a filled line representing the heat transfer from the heat belt to plant 2 and a branch bypassing plant 2. This branch is mixed with the heat-belt cold branch receiving heat from

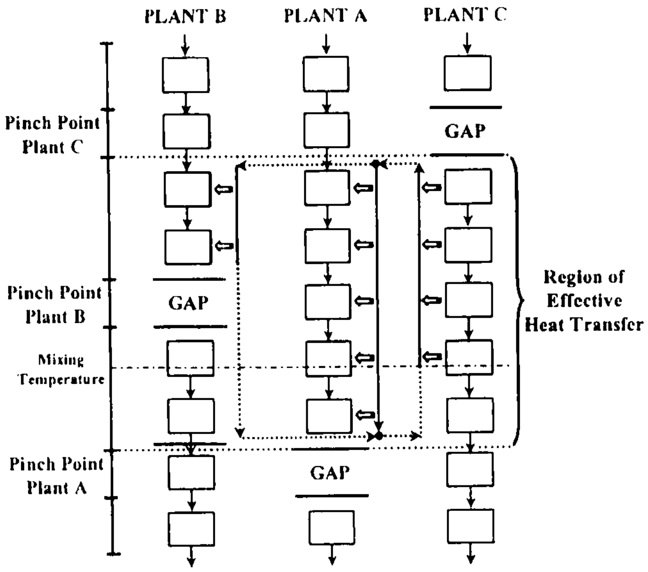

Figure 7. Heat belt for 2nd case.

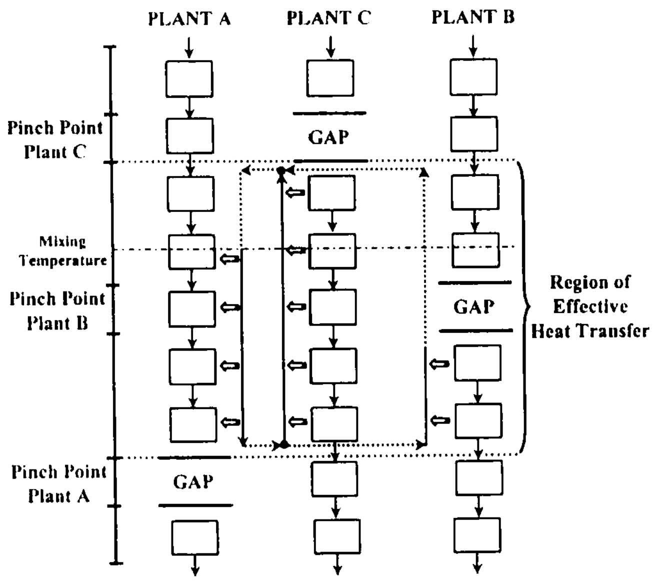

Figure 8. Heat belt for the 3rd case.

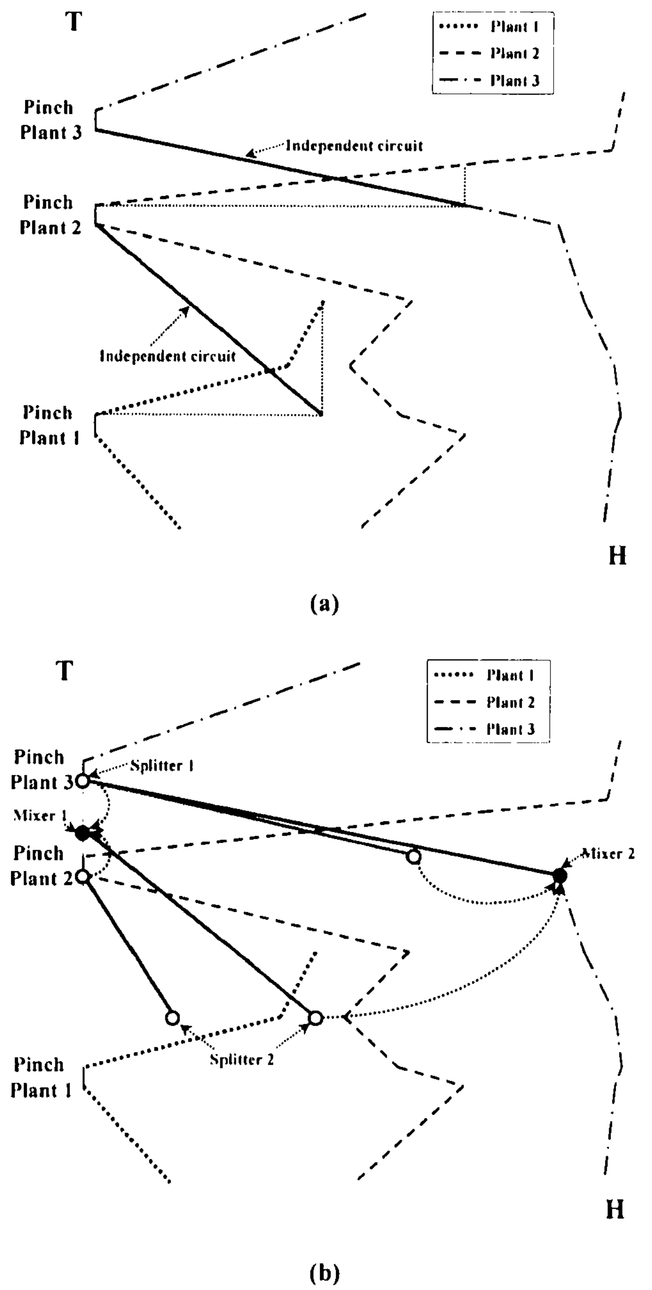

Figure 9. Composite curves for the 1st case.

plant 2 (mixer 1). The mixed stream (filled line) represents the heat-belt hot branch delivering heat to plant 1. This branch is then split in the heat-belt cold branch receiving heat from plant 2 and the stream bypassing plant 2 (splitter 2). The latter stream finally mixes with the heat-belt hot stream delivering heat to plant 2 to generate the heat-belt cold stream receiving heat form plant 3.

Example 1

This example was used in Bagajewicz and Rodera (2000) to show indirect integration among a set of three plants. The results of applying individual pinch analysis to each of the plants are shown in Table 1.

First, the plants are arranged in order of increasing pinch temperatures (1st case) and model **P1** is solved. Figure 10a shows the solution that transfers the maximum possible heat. As the flowrates bypassing the plant with the intermediate pinch temperature are zero, this solution is equivalent to the one obtained by Rodera and Bagajewicz (2000) using two independent circuits. If some flow is forced to bypass the plant in the middle (e.g. lower bound $F_2^U = 0.5$), a new solution is obtained. As Figure 10b shows, there is a decrease in the total savings. This is due to the mixing required to establish the heat belt. Notice the structural difference between this heat belt and the multi-operation circuit presented by Rodera and Bagajewicz (2000). The heat belt in Figure 10b is also capable of obtaining

Table 1. Individual plant pinch analysis for example 1.			
Problem	Pinch	Minimum Temperature	Minimum Heating Utility
Cooling Utility			
	(°C)	(kW)	(kW)
Test Case #2	90	107.5	40.0
Ciric & Floudas (1991)	200	600.0	2100.0
4sp1	249	128.0	250.0

savings if plant 2 is shut down. The presence of the bypass streams not used when the system is fully integrated makes this possible. The case in which the plant with the intermediate pinch temperature is located first in the sequence is now considered (2nd case). The solution of model **P1** is presented in Figure 11. Maximum total savings with this array are lower than in the 1st case (increasing pinch temperatures order), because only plant 4sp1 is delivering the heat to the heat belt. Finally, the case in which the plant with the intermediate pinch temperature is located last in the sequence is considered. The solution to model **P1** establishes a single circuit between the plant with the highest pinch temperature (4sp1) and the plant with the lowest pinch temperature (Test Case #2). The reason for the single circuit is that no heating utility savings can be attained in the plant with the intermediate pinch temperature (Ciric and Floudas, 2000).

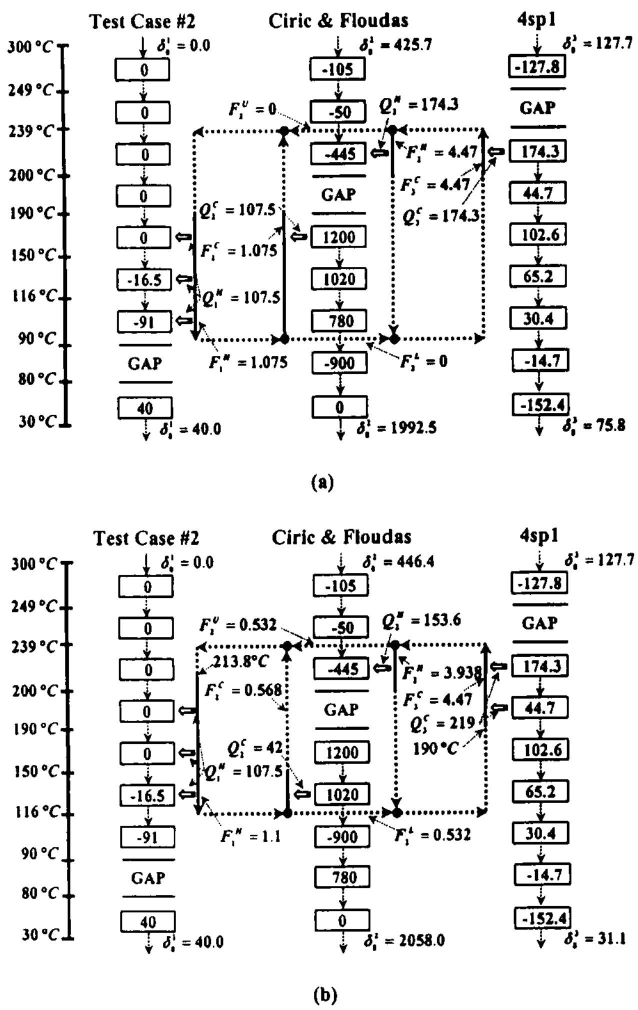

Figure 10. Solutions for the 1st case.

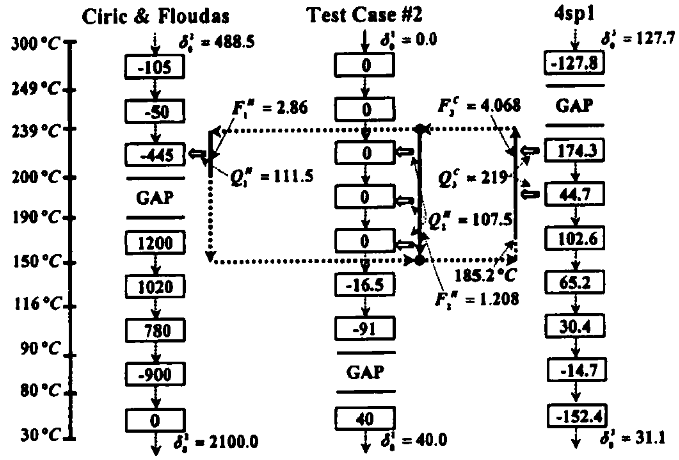

Figure 11. Solutions for the 2nd case.

Example 2

This example was introduced by Ahmad and Hui (1991) to show the application of their procedure to find required heat flows between what they called 'areas of integrity'. Table 2 shows the results of independently applying pinch analysis to each of the areas.

The presence of an assisted heat integration case is detected by the solution of the targeting model for energy savings applied to indirect integration (Bagajewicz and Rodera, 2000). This model considers independent transfer of heat within each interval and requires simultaneous parallel and opposite heat transfer between the areas. Because model **P1** only considers effective heat, lower savings are expected. Moreover, limitations in the amount that can be transferred arise when a single pipe is used to collect or deliver heat to the plants.

When model **P1** is solved for the case in which the plants are arranged in order of increasing pinch temperatures (1st case), the solution shown in Figure 12 is obtained. Similar to the previous example, the flowrates bypassing the plant with the intermediate pinch temperature are zero, and this solution is equivalent to the use of two independent circuits. The presence of the bypass streams is useful for the case in which area C is shut down. In the 2nd case the plant with the intermediate

Table 2. Individual plant pinch analysis for example 2.			
Problem	Pinch	Minimum Temperature	Minimum Heating Utility
Cooling Utility			
	(°C)	(kW)	(kW)
Area A	70	43	12.75
Area B	500	1.5	19.35
Area C	210	25	36.5

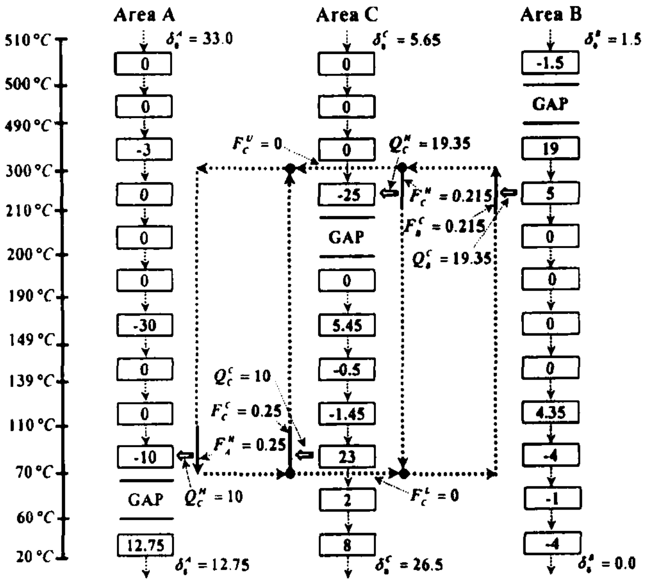

Figure 12. Solutions for the 1st case.

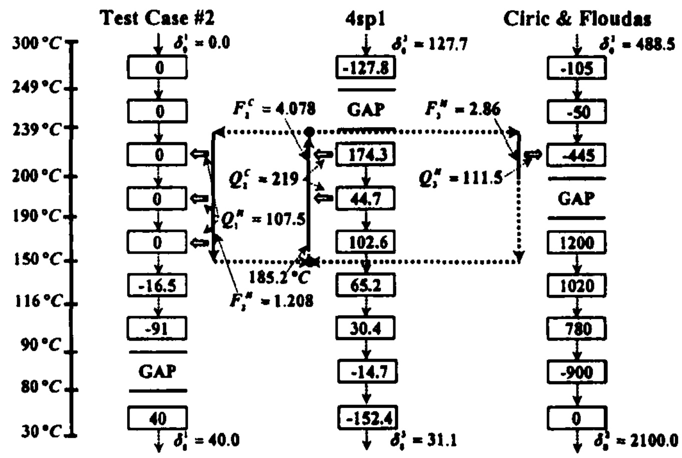

Figure 14. Alternative heat belt for the 3rd case array.

pinch temperature is located first in the sequence. Then, the solution to model **P1** establishes a single circuit between the plant with the highest pinch temperature (area B) and the plant with the intermediate pinch temperature (area C). Energy savings are limited in this case to the amount that area B can transfer. The final array locates the plant with the intermediate pinch temperature last in the sequence (3rd case). For this case, the solution to model **P1** is shown in Figure 13. In this case, mixing and the structural restrictions imposed by the location of the heat belt reduce the energy savings when compared with the solution shown in Figure 12.

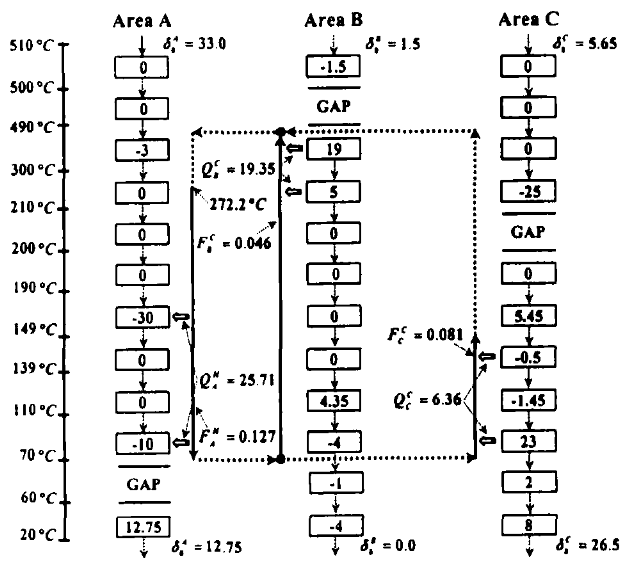

Figure 13. Solutions for the 3rd case.

Alternative Heat Belt Representations

Consider the location of the plants of example 1 shown in Figure 14. Since the plant with the intermediate pinch temperature is located last (3rd case), this plant cannot receive efficient heat. However, consider the inclusion of a hot branch for this plant and the possibility of this branch of transferring heat from left to right. The heat belt can now deliver heat that is collected from the with the intermediate pinch temperature if the direction of the lines going from this plant to the plant with the highest pinch temperature is reversed. The same savings are obtained as the result shown in Figure 10. It is not possible to obtain this solution by using model **P1**, as it represents the instance of another type of heat belt. However, model **P1** first can be solved using the arrangement of Figure 11, and then the solution can be used to establish the circuit of Figure 14.

Conclusion

The new concept of the heat belt that indirectly integrates a system of n plants by using an intermediate fluid was introduced. Some characteristics of this circuit were discussed. An MINLP for the special case of three plants was formulated. Replacement of binary-continuous bilinear terms by a linear set of equations is possible. The resulting MILP problem is therefore solved. Examples that show the utility of the heat belt for different relative locations of the three-plant system were presented. Future work involves the solution of assisted cases as well as the extension of the model to a larger number of plants.

Appendix 1

This appendix reviews the basic concepts related to heat integration across plants in the total site. Consider a set of n plants sorted from left to right in order of increasing pinch temperatures for which minimum utility targets have been calculated independently. Figure A1 taken from Rodera and

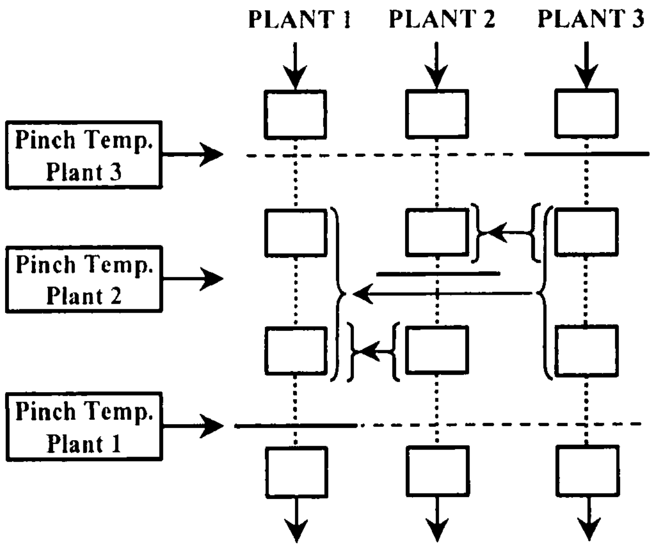

Figure A1. Effective heat transfer.

Bagajewicz (1999a) shows this for the case of three plants. When any two plants of the set are taken into account, the region between pinch temperatures is the region in which effective transfer leading to utility savings takes place, because the plant with the higher pinch temperature is a net heat source and the plant with the lower pinch temperature is a net heat sink. This type of heat transfer is called 'effective heat transfer' (Rodera and Bagajewicz, 1999a).

The instance in which maximum energy savings are attained solely by transferring heat in the region between pinch temperatures was defined by Rodera and Bagajewicz (1999a) as 'unassisted heat transfer case'. Moreover, the authors showed that to attain maximum energy savings in certain cases, effective heat transfer across two plants (i.e., taking place between pinch temperatures) must be accompanied by assisting heat transfer in the reverse direction and outside the region between pinch temperatures. This instance is referred as 'assisted heat transfer case'. The interaction among plants becomes more complex for the case of more than two plants. In this case, assisting heat does not have to come from the same plant that is receiving the heat, as it can be supplied by other plants. Bagajewicz and Rodera (2000) consider examples in which assisted heat transfer requires assisting heat in the inverse and in the same direction as the effective heat.

Appendix 2

The formulation of Equations (4) and (5) is presented in this appendix and is based on equivalent equations introduced by Rodera and Bagajewicz (1999a). The starting point is to consider the already defined binary variables that determine the optimum upper and lower limits in the temperature intervals in which the hot branch transfers heat to plant j (the analysis is similar for the cold branches for which similar binary variables were already defined):

$$Y_{ij}^{UH} = \begin{cases} 1 & \text{Internal } (i{+}1) \text{ is the upper temperature interval} \\ 0 & \text{Otherwise} \end{cases}$$

$$Y_{ij}^{LH} = \begin{cases} 1 & \text{Internal } i \text{ is the lower temperature interval} \\ 0 & \text{Otherwise} \end{cases}$$

To guarantee that only a temperature interval is the upper/lower one in plant j, the following inequalities are introduced:

$$\sum_{i=0}^{m-1} Y_{ij}^{UH} = 1 \qquad \text{(A1)}$$

$$\sum_{i=1}^{m} Y_{ij}^{UL} = 1 \qquad \text{(A2)}$$

In addition, continuous variables Z_{ij}^{H} that were already defined are use to determine those temperature intervals in which the hot branch spans. These variables are related to Y_{ij}^{UH} and Y_{ij}^{LH} by the following equalities:

$$Z_{ij}^{H} = Y_{(i-1)j}^{UH} \qquad i = 1 \qquad \text{(A3)}$$

$$Z_{ij}^{H} = Z_{(i-1)j}^{H} + Y_{(i-1)j}^{UH} - Y_{(i-1)j}^{LH} \quad \forall i = 2,\ldots,m \qquad \text{(A4)}$$

These equalities are needed to restrict the values of the heat transferred from the intermediate fluid to plant j to be zero for temperature intervals that are not serviced by the hot branch. The restriction is imposed in those temperature intervals by using the following inequalities:

$$q_{ij}^{H} - U_{ij}^{H} Z_{ij}^{H} \le 0 \qquad \forall i = 1,\ldots,m \qquad \text{(A5)}$$

As an example, consider a hot branch for which the upper limit temperature interval is the first temperature interval of plant j. Then:

$$Y_{0,j}^{UH} = 1 \quad \text{and} \quad Z_{1,j}^{H} = Y_{0,j}^{UH} = 1 \qquad \text{(A6)}$$

Otherwise, $Y_{0,j}^{UH}$ and the first interval of transfer will be located at a lower level in the cascade of plant j. That is $Z_{1,j}^{H} = 0$. Let's consider now that the third temperature interval is the upper limit one. Then:

$$Y_{2,j}^{UH} = 1 \quad \text{and} \quad Z_{3,j}^{H} = Z_{2,j}^{H} + Y_{2,j}^{UH} - Y_{2,j}^{LH} = 0+1+0 = 1 \qquad \text{(A7)}$$

In any case, Y_{ij}^{LH} must be zero in the temperature interval in which Y_{ij}^{UH} is one in order for the hot branch to span at least a temperature interval. Finally, consider that the circuit ends in the fifth temperature interval. Then:

$$Y_{5,j}^{LH} = 1 \text{ and } Z_{6,j}^{H} = Z_{5,j}^{H} + Y_{5,j}^{UH} - Y_{5,j}^{LH} = 1+0-1 = 0 \qquad \text{(A8)}$$

Therefore, there will not be a transfer in the sixth temperature interval.

Nomenclature

F_{jk} product of heat capacity and flowrate for the intermediate fluid going from plant j to plant k, (kW/°C)

F_j^C product of heat capacity and flowrate for the heat-belt cold branch receiving heat from plant j, (kW/°C)

F_j^H product of heat capacity and flowrate for the heat-belt hot

	branch delivering heat to plant j, (kW/°C)
F_j^L	product of heat capacity and flowrate for the lower heat-belt main line bypassing plant it, (kW/°C)
F_j^{LT}	product of heat capacity and flowrate for the lower heat-belt main line going from plant j to plant k, (kW/°C)
F_j^U	product of heat capacity and flowrate for the upper heat-belt main line bypassing plant j, (kW/°C)
F_{jk}^{UT}	product of heat capacity and flowrate for the upper heat-belt main line going from plant j to plant k, (kW/°C)
i	temperature interval
j, k	chemical plant
m	total number of temperature intervals
n	total number of chemical plants
P_j	chemical plant
Q_{jk}	total heat transferred from plant j to plant k via the intermediate fluid, (kW)
Q_j^C	total heat transferred from plant j to the heat-belt cold branch, (kW)
Q_j^H	total heat transferred from the heat-belt hot branch to plant j, (kW)
$q_{i,j}^C$	heat transferred from plant j to the heat-belt cold branch in interval i, (kW)
$q_{i,j}^H$	heat transferred from the heat-belt hot branch to plant j in interval i, (kW)
T_i	lower interval temperature, (°C)
$U_{i,j}^C$	upper bound in the amount of heat transferred from plant j to the heat-belt cold branch in interval i, (kW)
$U_{i,j}^H$	upper bound in the amount of heat transferred from the heat-belt hot branch to plant j in interval i, (kW)
X_j^C	enthalpy of heat-belt cold branch before receiving heat from plant j, (kW)
X_j^H	enthalpy of heat-belt hot branch before delivering heat to plant j, (kW)
Y_{ij}^{LC}	binary variable that establishes the lower limit temperature interval i for the heat-belt cold branch delivering heat to plant j
Y_{ij}^{LH}	binary variable that establishes the lower limit temperature interval i for the heat-belt hot branch delivering heat to plant j
Y_{ij}^{UC}	binary variable that establishes the upper limit temperature interval i for the heat-belt cold branch delivering heat to plant j
Y_{ij}^{UH}	binary variable that establishes the upper limit temperature interval i for the heat-belt hot branch delivering heat to plant j
Z_{ij}^C	continuous variable that determines whether or not there is heat transfer in temperature interval i for the heat-belt hot branch delivering heat to plant j
Z_{ij}^H	continuous variable that determines whether or not there is heat transfer in temperature interval i for the heat-belt hot branch delivering heat to plant j

Greek Symbols

ΔT_i	interval temperature difference, (°C)
δ_0^j	minimum surplus to the first interval of plant j, (kW)
δ_0	original minimum surplus to the first interval of plant j, (kW)
δ_i^j	heat cascaded from interval i in plant j, (kW)

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